

Marginal Cost and the Fiscal Impact of a Proposed Tuition Tax Credit in South Carolina

Prepared for

The South Carolina School Boards Association

and

The South Carolina Association of School Administrators

MARGINAL COST AND THE FISCAL IMPACT
OF A PROPOSED TUITION TAX CREDIT
IN SOUTH CAROLINA

I. INTRODUCTION

Much discussion has centered on the proposed tuition tax credit proposals currently being considered by the State's General Assembly (H. 4908 in 2004 and H. 3204 in 2005). Both versions of the proposal provide tax credits to qualifying persons who opt to either home school or send their children to private schools. In addition, they offer unlimited tax credits to corporations and individuals who donate funds to private school foundations. Part of the underlying argument put forth by the supporters for this concept is that parents would have a greater array of educational choices and that the competition created by this expanded choice will force public schools to provide their services in a more efficient and effective manner. There are arguments for both sides about the validity of these claims. No research conducted specifically for South Carolina was found to support these claims and thus they warrant extensive analysis. Most of these issues are beyond the scope of this analysis.

There is, however, one specific concern of the supporters of public education who oppose this concept. This is that it will create financial difficulties for schools that lose enrollment. The issue relates to the fact that State support for public education is tied to the number of students enrolled. Lower enrollments then lead to fewer State dollars and shrinking budgets to educate the students that remain. The crux of this issue is whether or not a school's costs decline sufficiently, due to lower headcounts, to offset this loss in State revenues. ***That is, the central question is whether costs will decline enough as students leave public schools to attend private schools, encouraged by the tax breaks, to offset declining State support?***

A paper prepared for the South Carolina Policy Council entitled "Marginal Cost of a Student" written by Professor Cotton Lindsay of Clemson University ("Policy Council Report") concludes that declining costs to public school districts will be more than enough to offset reductions in State revenues to districts resulting from cuts in State funding due to declining enrollment. This study concludes that schools that lose enrollment because of this new tax credit will actually be fiscally better off than they were before the program.

The Policy Council Report has a number of problems, which we believe are so serious that basing an important educational policy decision solely on such results would be a profound mistake. While it is frequently claimed that various voucher and tax credit programs would give individuals more choices, it is also important to understand the true costs of such decisions. If students who remain in public schools find themselves in

schools with declining budgets and resources, this impact must also be considered as a cost of the proposed tuition tax credit program. Likewise, if local governments must raise taxes to offset reduced funding from the State, the program will merely shift the tax burden from one group of taxpayers to another group. Again, this is a serious policy issue and one that warrants extensive analysis.

While there are numerous issues surrounding this proposed change in South Carolina's educational policy, the objective of this report is simple and straightforward. ***The objective is to determine if the analysis of the Policy Council is sound enough to support the conclusion that the proposed tax credit plan would be financially beneficial to the State's public school system. That is, would schools with declining enrollments actually experience positive net financial impacts on their budgets?***

The highlights of this discussion are as follows:

- The SC Policy Council Report generates an estimate of the marginal cost of educating a student. The Policy Council Report argues that this estimated marginal cost would be the amount public schools would save when a student migrates from the public system to private schools because of the tax credit. Their report then argues that this savings more than offsets the loss in state funds associated with the departing student. Their argument is that public schools are then fiscally better off than they were before the student left.
- It is incorrect to apply the marginal cost concept as used in the Policy Council Report to project "savings" to the public schools because of the nature of the educational production process (indivisible or lumpy inputs).
- There also are substantial technical problems with the Policy Council Report's statistical analysis that undermine the validity of the report's estimation of the marginal cost of educating a student in South Carolina and its conclusion that school districts will be better off financially.

The following paper has two major sections. The first questions whether or not the use of the marginal cost concept is appropriate for the issue at hand. This question is examined from a conceptual vantage point and then from a real world or empirical point of view. The second part describes the technical problems and shortcomings of the Policy Council's statistical analysis. Here the concern is with the modeling and interpretation of results. Much of this material is based on the work of Tomonori Ishikawa, which is included as Appendix A. Mr. Ishikawa received a B.S. in Economics from MIT, has done doctoral work in Economics at Northwestern University, and is currently a Ph.D. student in the Department of Statistics at USC. He has extensive teaching experience in Economics, Finance, Strategy and Statistics, and is a Graduate Research Assistant for the South Carolina Education Oversight Committee.

II. IS MARGINAL COST THE CORRECT MEASURE?

The essence of the SC Policy Council Report and the argument for the adoption of a tax credit/voucher type program in South Carolina is based on a much-used concept in economics known as “marginal cost”. This is defined as the change in total cost for a producer (school) if it produces one more unit of output (student). The report then argues that if a student “migrates” from a public school to a private school, the public school would “save” this amount or that its decrease in cost would equal this marginal cost value. The offsetting factor for the school’s budget is that it would then lose the State “per-pupil” contribution resulting from the student’s departure. The report then posits that if the savings in cost (measured by marginal cost) is greater than the decrease in State funding, then the school is actually better off fiscally because of the departure of this student.

The Policy Council Report estimates what it refers to as a cost function based on data from over 1,000 public schools in South Carolina. There is, however, an important issue of the appropriateness of the “marginal cost” approach to the question at hand. Economists are trained to look at questions through the glasses of marginal analysis. The concept is taught in beginning economics classes and used throughout the economics literature. ***But, does the application of this concept hold when applied to this question— a significant public policy decision about public education?***

First, it is true that given the proposed tax credit plan, schools would lose the State’s per-pupil contribution for the departure of even one student. The Policy Council Report estimates an average of \$5,839 lost State revenue to a district per migrating student. However, in reality, the amount of this loss depends upon the fiscal conditions in a particular school district. These conditions include, among others, the State funding formula, the local tax effort, the district’s tax base and the district’s index of taxpaying ability. Thus, poorer districts are at risk of losing a larger amount than more wealthy districts where the State share is smaller and the local share is higher.

For example, for 2002-03 Horry County School District received \$938 per student in State EFA funding while Hampton 2 School District received \$1,641 per student in State EFA funding. Obviously, the fiscal impact on these two districts from the implementation of the proposed Tuition Tax Credit would be drastically different. ***Extending this to districts throughout the State, it is apparent that the financial impact of the proposed law would not be uniform on all districts, but would impact each district differently. The impact of this issue needs further and careful study.***

The Policy Council Report’s regression results indicate an estimated marginal cost for South Carolina of \$6,422 in 2005-2006, which the report states would be resources “freed by a migrating student in that year”. It is the roughly \$600 difference in lost State revenues and the “savings” estimated by the marginal cost (\$6,422 vs. \$5,839) which is highlighted as the “benefit” to public schools of losing students because of this new law.

Even assuming these estimates of marginal cost are correct and that the regression model is appropriate, there remains a substantial problem with the concept of using marginal cost in this manner. ***Is it reasonable to assume that a school will actually save \$6,422 from the departure of one student? Does the fact that the number of students in each class declines by one (say from 15 students to 14 students) actually result in any cost savings to the school in the real world, much less \$6,422?***

The Policy Council's argument has serious and fundamental flaws and is difficult to accept. While there are potentially some minor cost savings in the long run, such a small change in enrollment does not lead to fewer teachers or other personnel or reductions in other major components of per pupil cost. If the number of teachers and other personnel is not reduced, the potential cost savings due to a reduced classroom headcount are extremely limited. For example, based on 2002-03 data, personnel salaries and benefits represent 77% of total costs for an average school in South Carolina. In that same year, the Policy Council Report asserts that the total cost per student (State and local sources) was \$9,130, thus, salaries and benefits would represent 77% of this, or roughly \$7,043. ***It is difficult to understand how a school's costs are going to be reduced by \$6,400 when one student leaves when more than \$7,000 in per student costs are not variable in the short-run.***

The problem of applying this marginal cost model to school costs is one of indivisibilities. ***Unless the scale of such migration is significant enough to eliminate entire classes (and the teachers assigned to those classes) and perhaps programs, the exact effect of migration on cost saving is extremely doubtful.*** The impact of such major reductions in programs and classes on the quality of education is an entirely different but equally important issue. As noted in Appendix A (pages 5 and 6), "quasi-fixed costs" are ignored. Thus, even teachers become "fixed costs" when one or a few students leave a public school.

To conclude, it is a certainty that schools will lose State funds for each public school student that migrates to private school. The real question is whether there are any offsetting cost savings and if so, are the cost savings large enough to offset decreased revenues. ***Given the above observations, use of marginal cost estimates to answer this question is suspect.*** It would seem that additional study of this issue should be undertaken prior to the General Assemble making a decision of this magnitude.

III. STUDENT MIGRATION OVER TIME: A MORE REALISTIC ESTIMATE OF THE IMPACT ON DISTRICTS, SCHOOLS, GRADES AND CLASSES

The above section argues that it is difficult to support a claim that small declines in public school student enrollments would lead to the levels of cost savings to schools implied by the marginal cost estimate of some \$6,400 per student. It is further argued that cost savings could potentially come with large or stepwise (“lumpy”) declines in enrollment, which would allow entire classes to be canceled so that there could be a corresponding reduction in teaching and service staff. This section examines this issue by looking at the estimated migration rates out of public schools to private schools resulting from the proposed tax credit as presented in the Policy Council report but from a more “real world” perspective.

With the program’s initial phase-in year in 2005-2006 (full implementation in 2009-2010); the Policy Council Report estimated annual migration growing from 8,707 students in the first year, to 42,625 students departing public schools by 2009-2010.¹ In the aggregate, it would thus appear that a substantial number of students would be diverted from public schools as a result of the tuition tax credit incentive when the program is in place and available to all who qualify. However, this is somewhat misleading.

These annual estimates are shown in the first column of Table I. As noted, the figures are statewide estimates and are based on the Policy Council’s mid-level migration rates.² The remaining columns contain estimates of what these migration levels would be on the average school district -- on a per school, per grade level, and per class basis. Thus, during the first year, the average school district would experience 102 students leaving public schools as a result of the program. This breaks down to about 15 students per school, which is about 3 students per grade level in each school which is equal to about 1.5 students per class room. Five years later, the average decline per school district grows to just over 500 students, which is equivalent to about 39 students per school, 10 per grade and 5 per classroom.

The question is whether such public school enrollment changes are significant enough to reduce school costs by an amount equaling marginal costs times the number of departing students. For example, using the report’s figures, the average school district would supposedly “save” nearly \$600,000 in the first year of implementation. With an average reduction of less than two students per classroom, it is doubtful that the average

¹ See tables 3a-3f in *Fiscal Impact of the Universal Scholarship t/Tax Credit Proposal*, Cotton M. Lindsay, A report to The South Carolina Policy Council In cooperation with The Lead Foundation, March 2004

² See tables 3a-3f in *Fiscal Impact of the Universal Scholarship t/Tax Credit Proposal*, Cotton M. Lindsay, A report to The South Carolina Policy Council In cooperation with The Lead Foundation, March 2004. First year migration rates range from 5,079 to 12,334 students. The mid-level migration rate is 8,707 in the first year.

school would see any reduction in costs. However, due to the lower headcount, the district would receive about \$300,000 less in State allocations. Statewide, the loss in State funds for public schools would total over \$26 million – with no demonstrated reduction in school costs.

The impacts from such a policy change would be cumulative. For example, as seen in Table 1, in the second year of implementation, the average district would lose another \$515,000 in state funding, but see no reduction in costs due to a reduction in headcount of an average of two students per classroom. Therefore, in the first two years, the schools may again not have experienced reductions in headcounts large enough to reduce costs, but the average district would have lost over \$800,000 in State funding. Statewide, the public school system would have seen its funding cut by almost \$70 million with the possibility of no discernable reduction in costs.

At full implementation, state funds would have been reduced annually by over \$350 million. In the year 2009-2010, the report estimates a decline of less than 40 students per school, 10 students per grade and 5 students per class. The conclusion that a decline of ten 4th graders (less than a full class of students) in a typical elementary school would reduce costs by nearly \$60,000 is without any solid basis in economic reality and is difficult to believe. At best, the estimated migration per school in the last year is 39 students. But, this number is spread over several grades and is unlikely to result in the need for fewer classes and teachers unless the students are concentrated in a particular grade. This assumption ignores core requirements and State law requiring certain courses be taught in certain grades, etc.

In conclusion, Table I and the SC Policy Council Report's own estimates provide very little evidence to expect the scale in student movement at the school or grade level that would cause quasi-fixed costs to decline in any significant manner. The annual averages per school, per grade and per class are quite small. Further, even when viewed as a cumulative process, it still takes several years before what might be considered as even minor cost savings to schools become possible. However, during this cumulative time period, public schools would suffer losses of millions of dollars in State funds with little or no actual reductions in costs.

Table 1

**Analysis of Fiscal Impact of Tuition
Tax Credit**

2005-06	State Total	Per District	Per School	Per Grade	Per Classroom
Policy Council's Estimated Student Migration	8,707	102	15	4	2
Policy Council's Estimated Savings	\$ 50,837,472	\$ 598,088	\$ 84,729	\$ 21,182	\$ 10,591
Reduction in State Funds to Districts	\$ 26,119,612	\$ 307,290	\$ 43,533	\$ 10,883	\$ 5,442
Actual Savings Realized by Districts	\$ -	0	0	0	0
Actual Loss in Funds to Districts	\$ (26,119,612)	\$ (307,290)	\$ (43,533)	\$ (10,883)	\$ (5,442)
2006-07	State Total	Per District	Per School	Per Grade	Per Classroom
Policy Council's Estimated Student Migration	14,611	172	24	6	3
Policy Council's Estimated Savings	\$ 85,311,992	\$ 1,003,670	\$ 142,187	\$ 35,547	\$ 17,773
Reduction in State Funds to Districts	\$ 43,832,159	\$ 515,672	\$ 73,054	\$ 18,263	\$ 9,132
Actual Savings Realized by Districts	\$ -	0	0	0	0
Actual Loss in Funds to Districts	\$ (43,832,159)	\$ (515,672)	\$ (73,054)	\$ (18,263)	\$ (9,132)
Cumulative Loss in State funds since 2005-06	\$ (69,951,771)	\$ (822,962)	\$ (116,586)	\$ (29,147)	\$ (14,573)
2007-08	State Total	Per District	Per School	Per Grade	Per Classroom
Policy Council's Estimated Student Migration	21,580	254	25	6	3
Policy Council's Estimated Savings	\$ 126,003,055	\$ 1,482,389	\$ 144,003	\$ 36,001	\$ 18,000
Reduction in State Funds to Districts	\$ 64,738,682	\$ 761,632	\$ 73,987	\$ 18,497	\$ 9,248
Actual Savings Realized by Districts	\$ -	0	0	0	0
Actual Loss in Funds to Districts	\$ (64,738,682)	\$ (761,632)	\$ (73,987)	\$ (18,497)	\$ (9,248)
Cumulative Loss in State funds since 2005-06	\$ (134,690,453)	\$ (1,584,594)	\$ (190,573)	\$ (47,643)	\$ (23,822)
2008-09	State Total	Per District	Per School	Per Grade	Per Classroom
Policy Council's Estimated Student Migration	30,753	362	32	8	4
Policy Council's Estimated Savings	\$ 179,568,136	\$ 2,112,566	\$ 184,172	\$ 46,043	\$ 23,022
Reduction in State Funds to Districts	\$ 92,259,703	\$ 1,085,408	\$ 94,625	\$ 23,656	\$ 11,828
Actual Savings Realized by Districts	\$ -	0	0	0	0
Actual Loss in Funds to Districts	\$ (92,259,703)	\$ (1,085,408)	\$ (94,625)	\$ (23,656)	\$ (11,828)
Cumulative Loss in State funds since 2005-06	\$ (226,950,157)	\$ (2,670,002)	\$ (285,199)	\$ (71,300)	\$ (35,650)
2009-10	State Total	Per District	Per School	Per Grade	Per Classroom
Policy Council's Estimated Student Migration	42,625	501	39	10	5
Policy Council's Estimated Savings	\$ 248,888,292	\$ 2,928,098	\$ 226,262	\$ 56,566	\$ 28,283
Reduction in State Funds to Districts	\$ 127,875,471	\$ 1,504,417	\$ 116,250	\$ 29,063	\$ 14,531
Actual Savings Realized by Districts	\$ -	\$ -	\$ -	\$ -	\$ -
Actual Loss in Funds to Districts	\$ (127,875,471)	\$ (1,504,417)	\$ (116,250)	\$ (29,063)	\$ (14,531)
Cumulative Loss in State funds since 2005-06	\$ (354,825,628)	\$ (4,174,419)	\$ (401,449)	\$ (100,362)	\$ (50,181)

Notes:

- Assumes .48 migration rate from SC Policy Council Report
- Assumes SC Policy Council's \$5,839 per student marginal costs savings
- Assumes \$3,000 per student state funding reduction
- # of schools assumed in 2005-06 = 600, 2006-07 = 600, 2007-08 = 875, 2008-09 = 975, 2009-10 = 1,100
- # of grades per school assumed in 2005-06 through 2009-10 = 4
- Elementary Schools = 600
- Middle Schools = 275
- High Schools = 225
- Assumes 15 to 1 Student/Teacher ratio
- Assumes 2 classes per grade
- Average Teacher Salary including fringe benefits = \$55,000
- Assumes 85 school districts
- * Total Migration rates -- See SC Policy Council's Tables 3a-3f

III. TECHNICAL PROBLEMS WITH THE ESTIMATIONS

Up to this point, this review and discussion has centered on using marginal cost estimates to measure savings to individual schools resulting from declining public school enrollment. It focused on the fact that many costs associated with the educational process are lumpy or quasi-fixed and thus, marginal costs as estimated by the Policy Council Report result in an overstatement of the cost savings to districts and schools from the anticipated declines in enrollment. It is next demonstrated that the calculated measure of marginal cost varies greatly with the particular specification of the model used. This raises the issue of the validity of the reported results.

In this and the following section, we turn to more technical issues associated with the estimation of marginal costs. That is, even if one were to accept the marginal cost approach, the Policy Council Report's modeling, estimating techniques, and interpretation are such that public policy decision makers should question the results as reliable values of the true marginal cost of public education in South Carolina. What follows is a "layman's summary" of these issues with the more technical details being presented in Appendix A.

The first concern that must be addressed is with how the "model" is constructed in that the factors used to "explain" educational costs are closely related to each other statistically. That is, each tends to mirror the other's movement. This creates a problem that econometricians refer to as "multicollinearity." This makes it difficult to separate out the influence of one explanatory variable (changes in enrollment) on the dependent variable (change in costs). In this particular case, the estimated relationships (e.g. marginal costs) are unstable and have large standard errors.

The instability is evident when one makes a small change in the model. For example when the study breaks down total enrollment into various sub-categories and estimates the model again, there are big changes in the other relationships (coefficients). This is very obvious from information included in the report but not discussed in the report. This evidence is found in Table 2 in Appendix A. Thus, depending on how one constructs the model, big differences in the important measure of marginal cost would be observed. The Policy Council Report has nothing to say about which model he estimates is "correct" but picks the first equation to derive the conclusion. There is no way to know why these findings are better than that resulting from any other specification of the model because of the instability. This raises questions about the reliability of the \$5,839 marginal cost figure. In fact, the weighted marginal cost estimate from his second regression gives a marginal cost of \$5,269, and instead of saving \$600 per migrating public school student, the schools end up losing about \$570 per migrating student (see page 11 of Appendix A).

The second technical issue related with the estimation of marginal cost has to do with what are referred to as standard errors. A standard error of an estimate is a measure of the accuracy of a prediction made with a regression. Large measured standard errors indicate that we should have little confidence that the estimated relationship between one explanatory variable and the dependent variable is close to the actual relationship. If large standard errors are present, then the estimates are so imprecise that it is difficult to rely on a traditional measure of whether or not the relationship is “different from zero”. In Appendix A, it is calculated that the standard error around the estimated fixed cost of \$292,899 is + or - \$1.25 million. With 0 being within a range of \$1.25 million (plus or minus) Professor Lindsay concludes that the estimate can not be shown to be different from zero and thus there are no fixed costs in the process. This is certainly contrary to a vast majority of the literature on education production and costs and is, in addition, counter-intuitive.

In fact, the study appears to misinterpret its own results on the fixed cost issue. Since the constant value for the first regression is deemed “statistically different from zero” in the first regression (Table 1a) it is concluded that there are no fixed costs. It fails to comment, however, on the middle and high school variables which have zero/one values. That is, for the middle school variable, it takes on a value of one if the school is a middle school, zero otherwise. The same is true for the high school variable. Both of these coefficients are statistically significant and positive. Being “shift variables”, they reflect the existence of fixed costs. This is discussed in Section 3.2 in Appendix A. This important point is not mentioned in the report.

IV. SENSITIVITY ANALYSIS

Another way to examine the appropriateness of the marginal cost results as evidence in support of any proposed tuition tax credit plan is to examine the margin of error of the estimated coefficient of the enrollment variable in the regression. A margin of error is a statistical measure which allows one to determine a range of values around an estimated coefficient where it is possible to state with a given level of confidence (e.g. 95%) that the actual value of the relationship rests within this range.

Thus, for the Policy Council’s report Table 1a results, the 95% confidence interval about the \$2,443 coefficient for Enrollment can be estimated as a plus or minus \$2,006. (See Appendix A, Section 4.3). This tells us then that we are 95% certain that the actual relationship between enrollment and cost lies within an approximate range of \$2,000 above and below the \$2,443. This is a very large range and raises serious questions about making important public education policy decisions on such weak analysis.

The lower end of this wide range is only \$416. Keep in mind that this number is still within the 95% range of confidence or probability. If this were indeed the case, the marginal cost of educating a student (and thus the savings a school would realize when a student migrates to a private school) in South Carolina would be \$3,793 and public schools would experience a \$2,046 **LOSS** for every student who left for private school because of the tax credit plan. Thus, we can say that such significant and negative budgetary impacts are definitely within the range of being possible if such a tax credit type of program were implemented in South Carolina. This raises concerns about the validity of the marginal cost estimate as presented in the SC Policy Council report.

V. CONCLUSION

The objective of this work is simple and straight forward. It focuses on a very small part of the broad and complex issues raised by the tax credit proposal. The proposed policy would provide various State tuition tax credits to parents who send their children to private schools or home school them. It is a subject that merits, indeed demands, serious research and discussion with all the facts and evidence being fully disclosed, analyzed, and understood.

One critically important part of this discussion is the fiscal impact on public school finances. It is being argued by the proponents of the tax policy that public schools will in fact be in better fiscal health after the implementation of the policy and the migration of public school students to private schools. This conclusion hinges on the assumption of a marginal cost value of education (the “savings” that public schools would realize) in South Carolina exceeding the per-pupil support provided to schools by the State of South Carolina. Thus, the supporters of this type of tax credit claim that a school would save more in costs than it loses in State revenues each time a student leaves the public school system and that the public school system would continue to improve fiscally as each additional student departs to a private school.

The evidence provided in this set of papers casts serious doubt on this conclusion. ***First, given the lumpy nature of educational costs and the quasi-fixed nature of even teaching costs, the simplistic use of per pupil expenditures for marginal cost values is not well supported or analyzed, misleading and simply not logical.*** The first two parts of this paper make this point in several ways.

Second, the empirical statistical work done to generate the marginal cost estimate used in the SC Policy Council Report has major deficiencies. ***It is thus extremely dangerous to make such a major policy decision based on results which could range from public schools being made better off to situations where they actually lose thousands of dollars per departing student.*** If the latter were the case, local governments would have to either raise taxes to off-set reduced funding from the State due to the tax credit program or cut educational programs. If higher local property taxes are the result, this would simply shift the tax burden from one set of tax payers to another set.

Finally, it is not the intent of this analysis to be pro or con on the issues of school tuition tax credits. Such a decision rests on the shoulders of the State's elected representatives. For them to make informed decisions, it is important that they be provided with sound and accurate information. It may be that the majority of the legislature believes the potential benefits of such a program outweigh the potential negative impacts on public school financing. ***If that is the case, the legislature needs to understand the magnitude of such effects before making any final decision that may impact public schools for many years in the future.***

It is important that decision makers realize the potential fiscal cost of making such a change in educational policy. It would be extremely unwise to rely on the estimates in the S.C. Policy Council Report to conclude that public schools would be financially better off because of the tax credit program.

VI. ADDITIONAL ISSUES THAT WARRANT FURTHER STUDY

As mentioned above, there are several important issues that merit serious study but are beyond the scope of this report. It is recommended that these issues be evaluated thoroughly before decisions are made on a tax credit program in South Carolina. While these issues are briefly itemized here, the brevity of the discussion should not minimize the importance of their potential impact.

1. The proposed tax credit is not a program for a small percentage of the State's residents who have low income. Based on South Carolina Department of Revenue data, 96% of all tax returns in South Carolina have "taxable income" of \$75,000 or less. The tax credit program would not be limited to a few – it appears that about 96% of all tax returns in South Carolina would be eligible. The SC Policy Council refers to the \$75,000 eligibility threshold as "income" rather than "taxable income". The Policy Council Report never refers to the limit as "taxable income". There can be tens of thousands of differences in "taxable income" and "income".

2. Related to this issue is that low-income families might not be able to take advantage of the proposed tax credit. For example, in order to take advantage of a \$3,000 income tax credit, a person would need to have a “taxable income” of around \$60,000.

3. In a broad sense, the State would have more students for which it would have financial responsibility but with less tax revenue. As demonstrated in this report, it is very unlikely that public schools would realize any “savings” as students left for private schools. However, it is certain that the State would have fewer tax revenues. Since the private school students whose education would be financed by tax credits get the full amount of tax credits, there could be less money (by the amount of the credits) for the students in the public K-12 system. In effect, the funds available for public K-12 students would be diluted because less State income tax would be collected.

4. A tax credit program’s fiscal impact on State revenues could be substantial. The SC Board of Economic Advisors has estimated the negative fiscal impact of a previous tax credit plan to be over \$234 million when fully implemented.³ This issue deserves very careful legal, economic, and financial analysis. For example, once fully implemented – even if the bill resulted in not a single student leaving the public system there would be roughly 65,000 students (estimated number of private school students as of 2004) in private and home schooling that could claim credits of at least \$3,200 per student. That would result in State income tax credits (i.e., less State revenue) of at least \$208 million --- with potentially no decrease in costs to public schools.

5. The amount of funds that will be contributed to Scholarship Granting Organizations is unknown. Even if these funds are not taken as credits by qualifying students, they will represent deductions from companies and individuals’ State tax liabilities and thus would reduce South Carolina tax revenues. The impact on State tax revenues of this aspect of the proposed law is not known. In fact, the Policy Council Report states that “Analysis of this behavior must contain a large element of speculation.”⁴ The report further states that “A tax credit has no such opportunity cost. It reduces tax liability by a dollar for each dollar contributed; hence it costs the taxpayer nothing to claim the tax credit for these donations. This takes us into the realm of whimsical rather than economic behavior.”⁵

Such a tax credit could be very costly to State revenues. A tax credit would have 20 times the impact on State revenues that a normal tax deduction. For example, it appears that if a company donated \$1 million to public education prior to this program, it would receive a \$1 million deduction from its SC tax liability. At 5%, such a donation would reduce State tax collections by roughly \$50,000. However, if the proposed law were passed, this same \$1 million donation (if made to a SGO) would be a credit, not a

³ South Carolina Board of Economic Advisors, Statement of Estimated State Revenue Impact, H.4908, May 11, 2004.

⁴ *Fiscal Impact of the Universal Scholarship t/Tax Credit Proposal*, Cotton M. Lindsay, A Report to The South Carolina Policy Council In Cooperation with The Lead Foundation, March 2004.

⁵ *Fiscal Impact of the Universal Scholarship t/Tax Credit Proposal*, Cotton M. Lindsay, A report to The South Carolina Policy Council In cooperation with The Lead Foundation, March 2004

deduction and as such would reduce State tax collections by \$1 million --- twenty times or \$950,000 more than the same donation as of now. At this time, it appears that no one has done adequate study of the potential fiscal impact of this aspect of the law.

6. Another area that has not had adequate analysis is the varying impacts from the proposed bill on different regions and districts within the State. This raises many profound issues similar to those addressed in the equity law suit.

7. To the extent that the State would be in essence subsidizing private schools and home schooling, the State would be paying for a new educational system that has no accountability for academic achievement. Schools and students in these private schools would not be subject to the Education Accountability Act or the federal No Child Left Behind Act.

8. The Policy Council Report includes students who have very high per pupil cost (for example, autistic students) in average per pupil cost, and then uses the average as the basis of the tax credit to fund the private or home school student who does not have the high cost characteristic. In effect, it uses the high cost of some students to inflate the amount that would be taken out of public school system for the benefit of less costly students.

9. The Policy Council Report uses as a basis for much of its analysis a compilation of cost data by Daniel Varat who analyzed data for 13 of the 85 districts. In addition to using such a small sample of districts, Varat's report makes several errors: (1) it includes the amount of construction bond issues as part of School District annual operating cost; (2) it double counts some revenue; (3) the report makes many errors in the actual number of students, in some years overstating them and in other years understating them, which has significant consequences when expressing cost per pupil or showing historical trends; and (4) it does not use actual audited financial figures, but uses consumer price index data to make adjustments in audited figures, and appears to overstate growth in costs. The reliance on this data base further erodes the validity of the Policy Council's tax credit analysis.

10. Would the proposed tax credit program in fact give parents a greater array of educational choices? Would a tax credit in fact create "competition" and force public schools to provide their services in a more efficient and effective manner?

All of these issues merit additional, thorough analysis.

Appendix A

Fixed Costs in Education: “The Marginal Cost of a Student” Revisited

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Executive Summary

Lindsay (2004) supports passage of the “Put Parents in Charge” legislation under consideration by South Carolina’s General Assembly. The study offers two regression estimates which conclude that: marginal costs dominate fixed costs in the provision of public education, and marginal costs are so high that public schools would be better off financially if students withdrew from the system.

After examining the issues framed by theories of cost analysis from managerial economics, this paper argues that schools face significant fixed costs which are misidentified in Lindsay. These fixed costs would not be reduced by lowered enrollments. Furthermore, while the withdrawal of headcount payments results from a transparent and automatic process, marginal cost savings are only *potential* cost savings. Schools would have to overcome many practical managerial challenges in order to realize any of those savings.

To highlight the unmet statistical challenges, this paper constructs two alternative calculations based on the figures provided in Lindsay. First, an alternative calculation of marginal costs indicates that schools would lose roughly \$570 per student lost. Second, “what if” calculations reveal that estimates of school cost savings are very sensitive to assumptions: considering a conservative estimate within the study’s sampling error produces a loss exceeding \$2040 to the school for each student lost.

This paper concludes that Lindsay (2004)’s marginal cost estimates are not plausible and reliable enough to inform policy decisions. Finally, a sensitivity analysis indicates that, instead of a boon to public school finance, the Put Parents in Charge legislation as constituted in Lindsay’s paper, is equally likely to cause serious fiscal harm.

1 Introduction and Overview

Lindsay (2004)¹ supports passage of the “Put Parents in Charge” Act under consideration by South Carolina’s General Assembly² by offering two regression estimates which conclude that: marginal costs dominate fixed costs in the provision of public education, and marginal costs are so high that public schools would be better off financially if students withdrew from the system. However, the study suffers from problems with its economic model and statistical methodology which raise questions about its suitability for policy analysis.

First, the economic theories of cost analysis in the paper place an over-emphasis on the role of marginal and variable costs in the provision of public education while neglecting the role of fixed costs. An heuristic evaluation of some of the factors of production that schools use to educate students suggests that the majority of all costs are indeed fixed and not variable or marginal.

Second, the statistical model used in the study introduces data problems which cause its estimates to be unstable with large standard errors. Furthermore, the study’s interpretation of the parameter estimates misidentifies fixed costs for middle and high schools while suggesting that fixed costs play a negligible role compared to marginal or variable costs in the provision of public education.

Third, this paper raises questions which are relevant for policymakers: Can schools really capture cost savings (if there are any)? Are marginal cost estimates reliable enough to inform policy decisions? What are the implications if the marginal cost estimates are not as expected?

Finally, using Lindsay’s own estimates in sensitivity analysis, this paper

¹Lindsay, Cotton (2004). The Marginal Cost of a Student. Columbia, SC: South Carolina Policy Council. Retrieved in Microsoft Word format from the URL: <http://www.scpolicycouncil.com/content/MarginalCost.doc>.

²The legislation was originally unveiled by South Carolina Governor Mark Sanford’s on February 26th, 2004, but did not pass the General Assembly in 2004. A modified version of the legislation has been pre-filed for consideration in 2005. Further details are available from the URL:

<http://www.scgovernor.com/interior.asp?SiteContentId=6&pressid=75&NavId=54&ParentId=0>

shows that instead of saving \$600 on average per migrating student, schools stand to lose between \$570 to \$2040 per migrating student.

When warranted, this paper provides detailed calculations supporting its conclusions in the appendix.

2 The Cost Structure of Schools

This paper begins by examining the cost structure of schools. Many costly resources are needed to provide an education to students. These resources are *inputs* or factors in the production of education. A cost analysis of schools begins by categorizing resources as either fixed or variable costs.

The importance of determining whether a cost is fixed or variable is that fixed costs do not fall from reductions in enrollment. All cost savings from reducing enrollments must necessarily come from shedding those inputs with variable costs—the resources with costs that vary with enrollment.

2.1 Fixed Versus Variable Costs in Education

Fixed costs do not vary with output. For schools, output is measured as enrollment, the number of students educated in a given school. In contrast, *variable costs* do change with enrollment. *Marginal cost* is a concept related to variable cost; it is the cost of educating the next student to enroll in a school.

By asking this question, one can determine whether a factor's cost is fixed or variable: "If one less student were enrolled in a particular school, could the school reduce the cost of that input?" If the school cannot reduce the cost of the factor in response to the loss of a student, then the factor is a fixed cost.

Consider some of the most common cost items in leadership, operations, instruction and instructional support:

1. Administrative salaries: If one less student were enrolled, would the school be able to reduce administrative costs? Generally, no. These salaries are fixed costs; schools would not change administrative salaries and overhead expense budgets if a student leaves the school.
2. Land, buildings and equipment: If one less student were enrolled, would the school be able to reduce its costs of maintaining land, buildings or equipment, or the rent associated with those assets? No. These maintenance costs are fixed; the school cannot lower these expenses if a student leaves the school.

3. Electricity: If one less student were enrolled, would the school be able to reduce its electrical bills? For the most part, no. Schools do not fractionally dim the lights and adjust the heating, ventilation and air conditioning systems when a student leaves the school. For the most part, even electricity is a fixed cost in providing education.
4. Transportation: If one less student were enrolled, would the school be able to reduce its transportation costs? No. School bus drivers drive set routes and are not compensated based on the number of students served by the bus route. Transportation is a fixed cost in providing education.
5. Teacher salaries: If one less student were enrolled, would the school be able to reduce the salary it pays to its teachers? In general, no. Schools do not pay a teacher less if they have 29 students in their class instead of 30.

An examination of these examples show that many of the costs of providing an education are fixed. These five items represent some of the most obvious costs associated with operating a school, and they are all fixed costs—they will not change if a school educates one less student.

2.2 Requirements to Achieve Potential Cost Savings

Schools cannot reduce fixed costs by losing a student here or there. A school only has the opportunity to reduce some of these costs if an entire classroom full of students leaves the school, or if the school could reorganize its instructional or administrative operations to eliminate a teacher or administrator. The amount spent on these costs do not vary smoothly as the enrollment of a school changes. They change in a *discontinuous* fashion, jumping up or down when enough students have joined or left the school to make a difference.³

Thus, in order to achieve cost savings from headcount reduction of the sort claimed in Lindsay, schools must have reduced headcounts in groups such that entire classrooms full of students can be eliminated to save costs. Changes to

³Economists refer to such “lumpy” or discontinuous costs as *quasi-fixed costs* when funds can only be spent or saved by the hiring or firing of a factor of production, not by varying how intensively that factor is used.

enrollments without changes to staffing and physical resource requirements will offer few opportunities to realize cost savings. Otherwise, classrooms may be a bit emptier, and teachers may find themselves teaching fewer students, but no money will have been saved.

In other words, a school saves no costs by reducing enrollments until it reaches a tipping point, when enough students of similar type have left the school so that the school can eliminate a teacher or administrator. At that point, the school's or district's administration must recognize the opportunity to save money by shedding a teacher or administrator and must implement disruptive changes to its instructional or administrative operations in order to realize the cost savings. No other process will allow the school to actually save money from student withdrawals.

2.3 Identification of Fixed Costs

The size of a school's fixed costs will vary according to its organizational level. Elementary schools have a more homogeneous student body than high schools. Therefore, elementary schools do not offer the diversity of activities and instruction that must be offered by a high school.

For example, high schools typically invite their students to participate in a range of activities such as sports, band or orchestra, academic quiz bowl teams, community service-oriented clubs and special interest clubs such as 4H, speech-debate teams, school newspaper and yearbook or "annual". These activities require teacher sponsorship, administrative support and facilities such as stadiums, practice fields, gymnasiums and dedicated activity rooms. These are all appropriately fixed costs items for high schools, but not elementary schools.

Aside from these co-curricular and extracurricular activities, high schools offer a wider range of instruction than elementary schools: advanced laboratory classes in biology and chemistry, as well as multiple tracks of instruction from remedial courses to Advanced Placement courses in math, the sciences, literature and history. Elementary schools simply do not offer these costly instructional programs.

Thus, in order to accommodate this diversity of activities and instruction, high school fixed costs should be greater than middle school fixed costs which

should be greater than elementary school fixed costs. However, Lindsay (2004) constructs a model that completely omits middle schools and high schools from the fixed cost estimates and fails to account for fixed costs increasing with organizational level.⁴

Fortunately, the study’s regression estimates, given in Table 1, can resolve this puzzle. By introducing indicator variables for middle schools and high schools, the regression equation shifts the intercept for these organizational levels. It is precisely this shifted intercept which represents a different fixed

Table 1: Components of fixed cost as reported in Lindsay’s Table 1a

Variable	Coefficient Estimate
Constant	292,899
Middle School	145,385
High School	656,933

cost at each organizational level. Thus, by appropriately adding these coefficients, as shown in Table 2, one can construct three distinct levels of fixed cost: one each for elementary, middle and high schools.⁵

Thus, once the coefficients have been properly combined, this paper confirms that fixed costs increase with the organizational level of students served. Elementary schools have the lowest fixed costs at \$292,899; middle schools have larger fixed costs at \$438,284, and high schools have the largest fixed costs at \$949,832.

⁴Lindsay (2004, page 7) notes the significance of the indicator variables, but does not make the connection with statistically significant fixed costs for middle and high schools:

It is interesting that the indicators for both middle and high school in this equation are positive and significant, indicating that both are more costly than elementary school. However, it is noteworthy that the intercept term, though positive, is insignificantly different from zero. This finding is consistent with the argument made above that we find no evidence of substantial fixed costs in the production of public schools.

⁵Appendix A details the rationale behind these calculations.

Table 2: Actual fixed costs by organizational level derived from Lindsay’s Table 1a

School Organizational Level	Actual Fixed Cost Estimates
Elementary Schools	\$292,899
Middle Schools	\$438,284 = \$292,899 + \$145,385
High Schools	\$949,832 = \$292,899 + \$656,933

3 Statistical Challenges

This paper continues with an examination of the statistical techniques used in Lindsay (2004).

3.1 Model Selection Issues

Lindsay (2004) constructs a multiple linear regression model of total costs by school. In the statistical model selected to estimate marginal cost, many of the explanatory variables are correlated with one another.⁶ This leads to two predictable difficulties: unstable parameter estimates and large standard errors.

The first impact of multicollinearity is unstable parameter estimates. In a well-constructed statistical model free of multicollinearity, small changes such as including or excluding a few additional explanatory variables shouldn’t change the importance of the remaining variables in the model. However, minor changes to Lindsay’s model (offered by the author himself) trigger major changes to the cost estimates.

The second impact of multicollinearity is large standard errors. This problem causes the inability to render sharp statistical judgments. Yet, Lindsay posits a definitive conclusion, proclaiming a “very different finding”⁷ that there are no fixed costs. Actually, substantial estimates of fixed costs are dwarfed by even larger standard errors.

⁶Econometricians refer to this well-known model selection problem as *multicollinearity*.

⁷Lindsay, page 4.

3.2 Unstable Parameter Estimates

In comparing Lindsay (2004)'s results in Appendix Tables 1a and 2a, many of the parameter estimates change in response to substituting (total) enrollment in Table 1a with enrollment disaggregated by student type (regular student enrollment, gifted student enrollment and special student enrollment) in Table 2a. Apart from this change, the other variables and the functional form of the regression equation remain the same in both tables, so this represents a minor change in model specification. However, this amendment to the list of included variables triggers significant changes to the estimates.

As shown in Table 3, the coefficient for the constant term, representing fixed costs for elementary schools, changes from \$292,899 in Table 1a to \$374,197 in Table 2a. The coefficients for the middle school and high school indicator variables, representing additional portions of fixed cost specific to middle and high schools respectively, change from \$145,385 and \$656,933 in Table 1a to \$66,477 and \$483,867 in Table 2a. In stable models, the change in the estimates of variables which are common to both regressions is close to zero.

Table 3: Unstable estimates due to model choice

Variables Common to Both Models	Coefficient Estimates		Change in Estimates (Should Be Close to Zero)
	Table 1a	Table 2a	
Constant	292,899	374,197	+\$81,298
Middle School	145,385	66,477	-\$78,908
High School	656,933	483,867	-\$173,066

Seemingly innocuous differences in estimates at the school level add up to multi-million dollar differences at the state level when fixed costs for all public schools in South Carolina are considered together.

3.3 Large Standard Errors

While it is usually thought that many costs in education are fixed, Lindsay (2004, page 4) announces the opposite conclusion:

Note also that our estimates cast doubt on the fixed cost aspect of this argument, as well. In standard cost function estimates the presence of fixed costs is indicated in the regression by a large and positive constant term. The finding of such a constant in the regression estimate indicates that substantial costs are present that are unresponsive to changes in output. As we illustrate in the appendix, our results suggest a very different finding.

It is true that “a large and positive constant term” in the regression would indicate the presence of substantial fixed costs. Lindsay’s Table 1a reports fixed costs estimates of \$292,899 on average per school. Leaving aside the criticism that this figure only represents fixed costs for elementary schools, the organizational level with the smallest fixed costs, the paper reports that this tidy sum is not different from zero—meaning that the study cannot distinguish the difference between \$292,899 and \$0. Therefore, the paper concludes, fixed costs are not statistically significantly different from zero.

However, formal statistical tests can find explanatory variables (like the intercept representing fixed costs here) to be insignificant for more than one reason. Specifically, in a regression equation of the form used in this model, a variable can be found insignificant for one of two reasons:

1. Its coefficient is actually close to zero.
2. Its coefficient is apparently large, but is so imprecisely estimated that the statistical test cannot distinguish the difference between it and zero.

Lindsay interprets the statistical test results as evidence of the first reason: fixed costs really are close to zero. That is, fixed costs of nearly \$300,000 are so small that they properly shouldn’t be distinguished from \$0. However, Appendix Table 1a in the report reveals that the statistical test results are a textbook example of the second reason: one finds a lack of significant fixed costs because the substantial estimates of fixed costs are dwarfed in comparison to very large standard errors.

3.4 Margins of Error

Readers unfamiliar with interpreting the results of regression studies can gain a measure of intuition from a more recognizable type of statistical study: a scientific survey poll taken to judge the intent of voters in an election. A common feature of statistically-valid polls⁸ is an explicitly-stated margin of error. A poll with a smaller margin of error is desirable because it is a more precise gauge of public sentiment than a poll with a larger margin of error.

This paper calculates margins of error for the estimated regression coefficients.⁹ If an estimate under study is within the margin of error away from another figure, then the result cannot be statistically distinguished from that figure. This is the essence of Lindsay’s interpretation of the significance of fixed costs: that \$0 is in a statistical deadheat with the estimate of \$292,899.

However, the margin of error around this estimate of \$292,899 is large: $\pm \$1.25$ million. It is true that zero is within $\pm \$1.25$ million of \$292,899, and one conclusion is that it is “statistically insignificant from zero.” But while it is technically correct to say that the estimate is insignificantly different zero, it is also valid to draw the conclusion that the estimate is insignificantly different from \$1.5 million—that figure is also within the margin of error.

With such large margins of error, these results are not precise enough to make definitive statements about fixed costs.

⁸Statistically-valid or scientifically-surveyed polls are designed using probability theory and incorporate measures of sampling error and confidence levels.

⁹Appendix C details this calculation.

4 Using Marginal Cost Estimates: Questions for Policymakers to Consider

The implications of technical difficulties in estimating the cost structure of a school are more than academic. This section considers three questions for policymakers analyzing the impact of the “Put Parents in Charge” Act: Can schools really capture cost savings (if there are any)? Are the marginal cost estimates reliable? What if the marginal cost estimates are not as expected?

4.1 Managerial Challenges: Can Schools Really Capture Cost Savings (If There Are Any)?

A single number at the center of Lindsay (2004)’s analysis of the fiscal impact of the Universal Scholarship Tax Credit proposal is the estimate of marginal cost. The accuracy of this number is critical to the claim that “public schools will save roughly \$600 more per student that migrates than the state will withhold in headcount payments.”¹⁰

This figure is the net result of two competing effects due to lower enrollments: lost payments from the state and saved costs. Both figures are estimates. The state bases its payments to schools on headcounts or enrollment.¹¹ Thus, the losses to schools are transparent. Schools will automatically receive less money from the state by reporting lower enrollments.

However, the ability of a school to capture gains from lower enrollments by saving costs that would have been spent to educate those students is far less obvious. It is worth examining whether such savings are possible or even likely.

A truism among educators is that educating one more child isn’t costly, but educating the 20th additional child is. Thus, a school can only reap the benefits of lower enrollments if the losses of students can be concentrated in a way such that the school can actually reduce its costly resources. For example, if a school loses a student, the school will only be able to capture savings

¹⁰Lindsay, page 4.

¹¹Anderson, Barton and Braman (2003) provides an excellent summary of the various public education funding mechanisms in South Carolina.

from losing that student if it can reorganize its instructional operations to fire a teacher or effectively remove a classroom from use. Such fixed costs are notoriously difficult to reduce.

Thus, in order to realize cost savings, if there are any, the school must recognize the potential savings, change its current organizational plans, and disrupt its operations to slash capacity (meaning fire teachers or staff and remove classrooms). Such actions represent significant managerial challenges for school and district leadership and will take time to evaluate and implement. They are not benefits that simply accrue to the school as enrollments decline.

This paper next examines whether the potential cost savings, represented by large marginal costs, exist at all.

4.2 An Alternative Calculation: Are Marginal Cost Estimates Reliable Enough to Inform Policy Decisions?

Lindsay (2004) presents the output of separate regression estimates in two appendix tables. In Appendix Table 1a, the estimated marginal cost due to total enrollment is \$2,422.95. However, another estimate for this figure can be derived from Appendix Table 2a which provides marginal cost estimates due to enrollment broken down by student type.

Using School Report Card data, total enrollment is disaggregated into enrollments by student type: gifted students, disabled students and the remaining “regular” students. Lindsay’s Table 2 provides the overall composition of these students in South Carolina. By using those percentages, another estimate of marginal cost due to enrollment can be calculated by taking a weighted average of marginal cost due to enrollment by student type.

One check on the reliability of marginal cost estimates would be to compare the marginal cost due to enrollment figures directly reported in Table 1a with those calculated from Table 2a. The numbers will not be numerically identical, but if the regression results are consistent, the figures should be roughly the same. Even if the figures are statistically equivalent (a likely result given the large standard errors), the figures should yield the same policy analysis:

schools will either save money under both estimated figures, or they will not. A policy-based measure for evaluating marginal cost estimates seems sound: does the evidence from Table 1a lead to the same conclusion as the evidence from Table 2a, or will the evidence lead to conflicting advice regarding the impact of relying upon these estimates?

Table 4: An alternative estimate of marginal cost

	Table 1a	Table 2	Table 2a
	Estimated	Composition	Estimated
	Coefficient	of Enrollment	Coefficient
		by Student Type	
Enrollment	\$2422.95		
Regular Student		74.7%	\$1443.60
Gifted Student		14.3%	\$244.50
Disabled/Special Student		10.9%	\$3940.80
Weighted Average (Should Match Enrollment Line Above)			\$1542.88

Using a weighted average, an alternative estimate of the resources freed by a migrating student is \$4019.49 now, or \$5268.73 in the first year of the program when “inflated”.¹² But, Lindsay notes that the state will withhold from the schools a payment of \$5,839 per student. So, instead of saving roughly \$600 per migrating student, schools will lose roughly \$570 per migrating student.

Table 1a provides evidence that schools will gain \$600 per student by shedding students from its rolls. The evidence from Table 2a yields a much different conclusion; schools will lose lose \$570 per student as enrollments shrink. The policy advice from these results is contradictory, and therefore, unreliable: Table 1a indicates that schools will fiscally benefit from lower enrollments, however, Table 2a implies that schools will be fiscally harmed by shrinking enrollments.

¹²Lindsay assumes a 7% per year growth rate for 4 years. This yields a total inflation factor just over 1.31.

4.3 Sensitivity Analysis: What Are the Implications If the Marginal Cost Estimates Are Not As Expected?

Sensitivity analysis is a technique for examining how the results of a calculation would change if underlying assumptions were other than expected. Many business analysts routinely conduct these “what-if” exercises to gauge the impact of adverse conditions that may be anticipated before funds are committed to undertake a project.¹³

Thus, sensitivity analysis using these results produces another method of evaluating claims of savings from headcount reduction. By using the study’s margin of error on enrollment alone, one can generate a conservative prediction for the enrollment-based estimate of marginal cost.

Using methods outlined in Appendix D.1, the margin of error on the estimated coefficient for enrollment in Table 1a is $\pm\$2006.10$. This yields a low estimate of the marginal cost due to enrollment alone, at the lower bound of the 95% confidence interval, of \$416.85. This generates a current conservative marginal cost estimate of \$2893.46. When inflated to 2005–06, this produces a marginal cost of \$3792.73 per student. After subtracting the lost headcount payment, schools would suffer a tremendous loss of \$2046.27 on average per migrating student.¹⁴

¹³For example, businesses routinely evaluate projects under consideration by performing net present value (NPV) calculations. This is a critical calculation because positive NPV projects will increase a firm’s value and create wealth for its shareholders, while negative NPV projects will reduce a firm’s value and harm its shareholders.

NPV calculations are based on estimates of future cash flows which can be attributed to the project and estimated future interest rates. A business analyst may conduct a sensitivity analysis of a project’s NPV by calculating the impact of alternative future cash flow estimates or different future interest rates.

Typically, prudent businesses pay careful attention to the potential effect of pessimistic assumptions and scenarios so that they can create contingency plans and calculate a project’s contribution to profitability under the worst conditions.

¹⁴Because the Education Finance Act mandates that less wealthy school districts receive higher payments, schools in poorer school districts would lose larger headcount payments than schools in wealthier school districts.

5 Conclusion

Lindsay (2004), “The Marginal Cost of a Student,” concludes that schools will benefit from reducing enrollments as cost savings exceed payment reductions. A central figure in this claim is the estimate of the marginal cost of educating a student.

The paper suggests that such marginal costs, and thus the potential savings “freed by migration,”¹⁵ are significant. This is in contradiction to a preponderance of established anecdotal and empirical evidence that large fixed costs, and not marginal costs, are the most important feature of school expenditures. In fact, the paper estimates that fixed costs for elementary schools, the organizational level with the smallest fixed costs, are roughly \$300,000. Yet these costs are so imprecisely estimated that statistical tests cannot distinguish this substantial figure from \$0.

The paper arrives at these conclusions based on an oversimplified application of economic and managerial theories and despite problematic choices in statistical model selection and estimation. Its parameter estimates are unstable, and standard errors are large. However, the difficulty posed by these technical shortcomings is more than academic. When policymakers use poor estimates for the basis of budget projections, the results are certain to be off target. Basing fiscal decisions on such unreliable figures could easily create further state and local budget deficits and render fiscal harm to schools as funds are withdrawn and savings are not realized.

This paper constructs two alternative estimates for marginal cost using Lindsay’s figures. One alternative estimate, calculated from a weighted average of the marginal costs of educating students of differing types, shows that schools will lose approximately \$570 per lost student. Another alternative estimate, derived from a “what if” calculation that considers estimates still within the sampling error of the study, but lower than expected, shows significant losses of over \$2,040 per lost student.

Finally, this paper has noted that Lindsay (2004) remains silent on the numerous practical managerial challenges upon school and district leadership that must be overcome in order to capture potential cost savings and translate them into benefits for a school’s bottom line.

¹⁵Lindsay, page 2.

The Universal Scholarship Tax Credit provisions of the “Put Parents in Charge” Act has the potential to greatly impact state and local fiscal policy with respect to educational finance in South Carolina. In addition, this paper has argued that the purported fiscal benefits at the school level are very much open to question.

What if more conservative estimates of marginal cost are considered? What will happen to funds at the school level? Instead of potential net savings of \$600 per student, using the same figures, this paper has shown the possibility of net losses of \$570 to \$2,040 per lost student. Even if the estimates of potential cost savings are not incorrect, will school and district administrators be able to identify and effect the changes needed to capture those savings?

Perhaps the worst consequence is the double jeopardy of fiscal harm. Faced with shortfalls from declining enrollment and the pervasiveness of fixed costs, schools would require more money, precisely at the moment that state and local coffers are emptied by the tax credit. If legislators and other policymakers fail to anticipate the impact of the foreseeable and likely negative consequences of such legislation, unfortunately, the onus and ultimate impact of fiscal harm will fall upon South Carolina’s children.



A Identifying Fixed Costs for Middle Schools and High Schools

In a regression study of costs, econometricians seek to estimate a cost function. The dependent variable is total cost and the explanatory variables are functions of output that vary with total costs.

Fixed costs are represented by the intercept of the cost function, typically by the coefficient on the constant term, indicated by the label “Constant” or “Intercept” in regression output tables. In fact, this is exactly what Lindsay (2004) interprets as fixed costs for schools of all organizational levels. However, Lindsay’s use of separate indicator variables for middle and high schools, shifts the intercept, and thus the fixed costs estimate, for those schools.

The sections below explain why the coefficient on the constant term only represents fixed costs for elementary schools.

A.1 Model Setup

In Lindsay’s model, C is total cost by school, Q is enrollment, S is teacher salaries, and M and H are indicator variables for middle schools and high schools respectively. ϵ is a random error term, typically assumed to be Normally-distributed with zero mean and constant variance. (Lindsay doesn’t specify.)

Thus, the population cost regression equation is:

$$C = \alpha + \beta Q + \gamma M + \delta H + \theta S + \phi QS + \epsilon \quad (\text{A})$$

M and H are indicator variables, so they take on the following values:

$$M = \begin{cases} 1 & \text{if the school is a middle school} \\ 0 & \text{otherwise} \end{cases}$$

and

$$H = \begin{cases} 1 & \text{if the school is a high school} \\ 0 & \text{otherwise} \end{cases}$$

The indicator variables for middle schools and high school function in a regression model by shifting the intercept of the regression line for schools at those organizational levels. Thus, fixed costs, represented by the intercept, will be different for high schools and middle schools compared to those for elementary schools.

This explanation of the model considers schools at all three organization levels, beginning with elementary schools as a starting point case.

A.2 Fixed Costs for Elementary Schools

Begin by considering the population regression line for an elementary school. $M = 0$ and $H = 0$ because an elementary school is neither a middle school (so $M = 0$) nor a high school (so $H = 0$).¹⁶

Plugging these values into the population cost regression equation (A) yields:

$$\begin{aligned} C &= \alpha + \beta Q + \gamma(0) + \delta(0) + \theta S + \phi QS + \epsilon \\ &= \alpha + \beta Q + 0 + 0 + \theta S + \phi QS + \epsilon \end{aligned}$$

The coefficients on the middle school and high school indicator variables disappear because they are multiplied by the zero values of their their variables. Thus, the population regression equation reduces to:

$$C = \underset{\substack{\uparrow \\ \text{intercept}}}{\alpha} + \beta Q + \theta S + \phi QS + \epsilon$$

¹⁶Some, but not all, schools with grades spanning multiple organizational levels may be considered using this particular model setup. If an elementary school also contains a middle school grade level, the value of the middle school indicator variable can be changed from $M = 0$ to $M = 1$. Similarly, if an elementary school also contains a high school grade level, the value of the high school indicator variable can be changed from $H = 0$ to $H = 1$.

This particular model setup can identify fixed costs for “K-12” schools including grades for all three organizational levels, but not schools that have both middle school and high school grade levels without any elementary school grade levels.

Based on 2003 Report Card data, 23 schools reported having both elementary and middle school, but no high school, grade levels together in the same school; 19 schools had both middle school and high school grade levels, but no elementary school grade levels; 4 schools had both middle school and high school grade levels, but no elementary school grade levels; no schools reported having only elementary and high school grade levels without any middle school grade levels.

This is the only interpretation of fixed costs offered in Lindsay (2004). However, the coefficient on the constant term only represents fixed costs for elementary schools alone:

$$\text{correct fixed cost estimate for elementary schools} = \hat{\alpha}$$

The next two sections show the existence of different fixed costs for schools of different organizational levels.

A.3 Fixed Costs for Middle Schools

For a middle school, $M = 1$ and $H = 0$. Plugging these values into the population cost regression equation (A) yields:

$$\begin{aligned} C &= \alpha + \beta Q + \gamma(1) + \delta(0) + \theta S + \phi QS + \epsilon \\ &= \alpha + \beta Q + \gamma + 0 + \theta S + \phi QS + \epsilon \end{aligned}$$

By grouping constant terms, the population cost regression equation for middle schools reduces to:

$$C = \underbrace{(\alpha + \gamma)}_{\text{intercept}} + \beta Q + \theta S + \phi QS + \epsilon$$

Therefore, due to the presence of indicator variables, fixed costs for middle schools are properly represented by the sum of the coefficient on the constant term and the coefficient on the indicator variable M , not the coefficient on the constant term alone:

$$\text{correct fixed cost estimate for middle schools} = \hat{\alpha} + \hat{\gamma}$$

A.4 Fixed Costs for High Schools

For a high school, $M = 0$ and $H = 1$. Plugging these values into the population cost regression equation and grouping constant terms as above, reduces the population cost regression equation for high schools to:

$$C = \underbrace{(\alpha + \delta)}_{\text{intercept}} + \beta Q + \theta S + \phi QS + \epsilon$$

Therefore, due to the presence of indicator variables, fixed costs for high schools are properly represented by the sum of the coefficient on the constant term and the coefficient on the indicator variable H , not the coefficient on the constant term alone:

$$\text{correct fixed cost estimate for high schools} = \hat{\alpha} + \hat{\delta}$$

A.5 Middle School and High School Fixed Costs Are Significantly Larger Than Elementary School Fixed Costs

Lindsay (2004)'s estimates of δ and γ , the coefficients on the middle school and high school indicator variables, are statistically significant.

Table A: Components of fixed cost from Lindsay Table 1a

Variable	Coefficient Estimate
Constant ($\hat{\alpha}$)	292,899 (0.46)
Middle School ($\hat{\gamma}$)	145,385 (2.81)**
High School ($\hat{\delta}$)	656,933 (9.89)**

The double asterisks marking the regression output reported by Lindsay indicates that they are statistically significant at the 2.5% level.¹⁷ (meaning they are larger than zero). Thus, there is ample statistical evidence to say that fixed costs for middle schools and high schools are significantly larger than fixed costs for elementary schools. ■

¹⁷Actually, these hypothesis tests should involve one-sided alternatives, that is, they should test whether the coefficients are zero or large positive numbers, and not whether they are possibly large negative numbers. Thus, they are actually more significant than reported by Lindsay. The t-statistic for middle schools has a p-value of 0.002523, and the t-statistic for high schools has a p-value of 0.

B Evidence of Multicollinearity in Lindsay (2004)

Coefficient estimates change wildly in response to a small change in the model's variables.

Table B: Comparing parameter estimates from Lindsay's tables 1a and 2a.

	Table 1a	Table 2a
Constant	292,899 (0.46)	374,197 (0.6)
Enrollment	2,422.95 (2.37)	–
Regular Student Enrollment	–	1,443.60 (1.4)
Gifted Student Enrollment	–	244.5 (0.23)
Special Student Enrollment	–	3,940.80 (3.38)
Teacher Salary	5.69792 (0.36)	4.514 (0.29)
Middle School	145,385 (2.81)	66,477 (1.16)
High School	656,933 (9.89)	483,867 (6.68)
Enrollment*Teacher Salary	0.06203 (2.46)	0.08456 (3.35)

As seen in Table B, the estimates change size and statistical significance:¹⁸

- The estimate of the coefficient on the constant term (what Lindsay interprets as fixed costs) changes by \$81,300 per school.
- Two of the three types of student enrollment lose statistical significance. (Only disabled student enrollment remains statistically significant.)

¹⁸For statistical significance, the table lists calculated t-statistics in parentheses below the coefficient estimates.

- The coefficient estimate on the middle school indicator variable changes by \$78,900 per middle school and loses statistical significance.
- The coefficient estimate on the high school indicator variable changes by \$173,000 per high school.
- The coefficient estimate on the enrollment-teacher salary multiplicative term changes by 0.02253. This changes Lindsay's marginal cost estimate by \$900.¹⁹



¹⁹Lindsay (2004)'s model estimates marginal costs as the sum of two components. The second component is the coefficient on the multiplicative term times average teacher salary. The study uses an average teacher salary of \$39,926. (See Appendix D.2 on page 25.) Thus, the impact on marginal cost estimates is:

$$0.02253 \times \$39,926 = \$899.53$$

C Margin of Error for Fixed Costs

This Appendix derives the margin of error for Lindsay (2004)'s fixed cost estimate.

Appendix Table 1a on page 9 reports an “absolute t-statistic” of 0.46:

Constant	292,899	“coefficient estimate”
	(0.46)	“t-statistic”

The relationship between the coefficient estimate and the t-statistic below in parentheses is:

$$\text{t-statistic} = \frac{\text{coefficient estimate} - 0}{\text{standard error of estimate}}$$

Thus, the standard error of the estimate is:

$$\begin{aligned} \text{standard error of estimate} &= \frac{\text{coefficient estimate}}{\text{t-statistic}} \\ &= \frac{292899}{0.46} \\ &= 636736.96 \end{aligned}$$

At Lindsay's 2.5% significance level (for a one-sided test), the margin of error (which corresponds to half the width of a 95% confidence interval) is:

$$\begin{aligned} \text{margin of error} &= \pm 1.96 * \text{standard error of estimate} \\ &= \pm 1.96 * 636736.96 \\ &= \pm \$1,249,445 \end{aligned}$$

So, the margin of error for this fixed cost estimate is a remarkable \$1.25 million. Lindsay (2004) argues that fixed costs are zero because the estimate of roughly \$300,000 cannot be distinguished from zero. In this context, zero is within the margin of error (meaning less than \$1.25 million away from \$300,000). However, \$1.5 million is also within the margin of error. Thus, an equally true statement is that fixed costs are \$1.5 million per school. ■

D Margins of Error for Sensitivity Analysis

This appendix derives a margin of error for the marginal cost estimate from a 95% confidence interval on the coefficient estimate on enrollment. Successive calculations use the lower bound of this confidence interval as a conservative estimate for sensitivity analysis.

D.1 A Conservative Estimate of Marginal Cost Due to Enrollments Alone

Appendix Table 1a on page 9 of Lindsay (2004) reports an “absolute t-statistic” of 2.37 for the estimate of the coefficient on enrollment:

Enrollment	2,422.95 (2.37)	“coefficient estimate” “t-statistic”
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Thus, the standard error of the estimate is:

$$\begin{aligned}\text{standard error of estimate} &= \frac{\text{coefficient estimate}}{\text{t-statistic}} \\ &= \frac{2422.95}{2.37} \\ &= 1022.342\end{aligned}$$

The margin of error, which corresponds to half the width of a 95% confidence interval, is \$2006.10:

$$\begin{aligned}\text{margin of error} &= \pm t_{1034,0.025} * \text{standard error of estimate} \\ &= \pm 1.96 * 1022.342 \\ &= \pm 2006.101\end{aligned}$$

Thus, a conservative estimate of the marginal cost to use as a pessimistic case in sensitivity analysis, is the coefficient estimate minus the margin of error:

$$\begin{aligned}\text{conservative estimate} &= \text{coefficient estimate} - \text{margin of error} \\ &= 2422.95 - 2006.101 \\ &= \$416.85\end{aligned}$$

Thus, the lower bound of the 95% confidence interval, and a pessimistic estimate of the marginal cost is \$416.85.

D.2 Using a Conservative Estimate in Sensitivity Analysis

Lindsay (2004) estimates marginal cost as a sum of two effects, one due to enrollments alone, and another due to the interaction between enrollments and teacher salaries. The report lists the overall marginal cost figure on page 4 and the coefficient values on page 9 in the appendix:

$$\begin{aligned}\text{marginal cost} &= (\text{MC due to enrollment alone}) + (\text{MC due to interaction}) \\ 4899.56 &= 2422.95 + 0.06203 \times \text{teacher salary}\end{aligned}$$

Thus, the value used for teacher salary can be recovered as:

$$\begin{aligned}\text{teacher salary} &= (4899.56 - 2422.95)/0.06203 \\ &= \$39,926\end{aligned}$$

Thus, one can ask in the context of a sensitivity analysis calculation, "What if the estimated marginal cost due to enrollments alone was lower than expected?"

A natural value to use as a pessimistic estimate to answer this question is the lower bound of the 95% confidence interval which this appendix has calculated as \$416.85. This value is just at the limits of sampling error for the estimated coefficient—it is not an *unreasonably* pessimistic figure from a statistician's perspective.

Changing just this figure in the equation yields a conservative estimate of the potential marginal cost savings of \$2,893.46:

$$\begin{aligned}\text{marginal cost} &= (\text{MC due to enrollment alone}) + (\text{MC due to interaction}) \\ &= 416.85 + 0.06203 \times 39926 \\ &= \$2893.46\end{aligned}$$

These calculations are based on 2001–02 estimates. In order to project the estimates to 2005–2006 in a way comparable to Lindsay (2004), the figures

are “inflated” by 7% per year. This means that the estimate is multiplied by a factor of approximately 1.31.²⁰ This yields an estimate of the marginal cost estimates in 2005–2006 of \$3,792.73:

$$\begin{aligned} \text{MC estimates in 2005–2006} &= \text{MC estimates in 2001–2002} \times \text{inflation factor} \\ &= 2893.46 \times 1.310795 \\ &= \$3792.73 \end{aligned}$$

But, Lindsay estimates the average “expected withdrawal from headcount reduction” as \$5,839 per student.²¹ Thus, a conservative estimate of the net savings from losing a student is $-\$2,046.27$:

$$\begin{aligned} \text{net savings} &= \text{potential MC savings} - \text{reduction in headcount payments} \\ &= 3792.75 - 5839 \\ &= -\$2046.27 \end{aligned}$$

In other words, by using a conservative estimate of the potential marginal cost savings from educating one less student, schools could plausibly *lose* nearly \$2,050 for every student it loses—a fiscally devastating result.

Aside from the issues related to forecasting budgets, this figure will vary from district to district as mandated by adjustment factors related to the Education Finance Act. For a student with the same characteristics, poorer school districts receive larger headcount payments from the state than a richer school district.

Thus, poorer school districts will lose more funding per student than average, and therefore, will be financially worse off than a richer school district under the same circumstances.



²⁰A growth rate of 7% per year compounded for 4 years yields a 31% gain:

$$1.07^4 = 1.310795$$

²¹This estimate depends upon budgetary processes, not statistical forecasts.

References

- [1] Anderson, Jo Anne, Melanie D. Barton and Amy L. Braman, “Funding Public Education: Issues of Adequacy, Equity and Sufficiency,” *Public Policy and Practice*, January 2003, pp25–31.
- [2] Lindsay, Cotton M. “The Marginal Cost of a Student,” mimeo, South Carolina Policy Council, 2004.